Estimating $\ln(5)$

a) Use the mean value theorem and the fundamental theorem of calculus to find upper and lower bounds on $\displaystyle\int_1^5 \frac{1}{x}\,dx$.

b) Compute $\displaystyle\int_1^5 \frac{1}{x}\,dx$.

c) Does your answer to (a) provide a good estimate of the value of $\ln(5)$?

7/8/25

Estimating $\ln(5)$

a) Use the mean value theorem and the fundamental theorem of calculus to find upper and lower bounds on $\displaystyle\int_1^5 \frac{1}{x}\,dx$.

b) Compute $\displaystyle\int_1^5 \frac{1}{x}\,dx$.

c) Does your answer to (a) provide a good estimate of the value of $\ln(5)$?

a)

$$\frac{F(b)-F(a)}{b-a} = F'(c)$$

$$\Rightarrow \quad \Delta F = F'(c)\Delta x = \frac{1}{c}(5-1)$$

$$\Rightarrow \min_{1<x<5}\left(\frac{1}{x}\right)\Delta x < \Delta F < \max_{1<x<5}\left(\frac{1}{x}\right)\Delta x$$

$$\frac{1}{5}(5-1) < \Delta F < 1 \quad (5-1)$$

$$\frac{4}{5} < \Delta F < 4 \quad \Rightarrow \frac{4}{5} < \frac{4}{c} < 4 \Rightarrow \frac{1}{5} < \frac{1}{c} < 1$$

$$\frac{\Delta F}{\Delta x} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\Rightarrow \quad \Delta F = \text{Average } F' \, \Delta x$$

$$\Rightarrow \min_{1<x<5}\left(\frac{1}{x}\right)\Delta x \le \frac{1}{5-1}\int_1^5 \frac{1}{x}dx\cdot(5-1) \le \max_{1<x<5}\left(\frac{1}{x}\right)\Delta x$$

$$\Rightarrow \quad \frac{4}{5} \le \int_1^5 \frac{dx}{x} \le 4$$

b) $$\int_1^5 \frac{1}{x}\,dx$$

$$= \ln|x| \Big|_1^5$$

$$= \ln 5 \approx 1.609$$

c) The lower bound $\frac{4}{5} \leq \ln 5$ may be useful but $\ln 5 \leq 4$ not so much.